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The generation of surface waves on an infinite cylindrical antenna by a point source is examined, and the amplitude of the leaky-wave or the Goubau-wave, depending on the reactive surface loading, is determined. A Fourier-time analysis then gives the surface-wave transient, and in both cases it is found to take the form of a precursor outside the light cone. It is concluded that both these surface waves are non-causal in character, and that this property is likely to be a general one for isolated modes that individually do not satisfy the radiation condition.

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SOME COMMENTS ON THE TIME-CAUSAL CHARACTERISTICS

OF LEAKY AND SURFACE WAVES

L. Lewin

Introduction

An excellent exposition of the nature and properties of leaky waves has been given by Hessel[1], including an extensive bibliography to which the interested reader is referred. As is now well-known, the field increases exponentially in the transverse plane at the same time that it attenuates axially. There is a certain angle, known as the critical angle, which defines a critical cone within which the radiation condition is satisfied. Outside the critical cone the field would increase on all directions, and it is clear that the leaky wave, in isolation, cannot exist as a realistic entity. Nevertheless it is a convenient abstraction with considerable use in leaky-wave antenna design. From a practical point of view we can distinguish three regions: a) very close to the source, where near-fields predominate, b) an intermediate region in which, close to the guiding structure, the field is predominately given by the leaky wave, and c) very far from the source, where the leaky wave has been exponentially attenuated so much that the source radiation (as modified by the presence of the guiding structure), with its inverse distance variation, dominates. On the critical cone the leaky wave merges with the source radiation and a clear distinction between them cannot be made. Outside the critical cone the actual radiation is that from the source, though the leaky wave, as a mathematical idealization, would, of course, completely dominate. A like set of three axial regions can be distinguished in an open lossy transmission line, like microstrip.

Somewhat different conditions prevail for the fields of a lossy dielectric-coated line, such as the Goubau line[2]. The wave is a slow wave, and in the absence of losses the field is radially attenuated. But to provide the energy absorbed in the lossy case the radial characteristics also involve an incoming radial wave feature, apparently in contravention of radiation requirements of outgoing waves at infinity. In a discussion of this aspect[3] it was indicated that the problem arose only because the surface wave was considered in isolation. If the radiation from the source is taken into account the surface wave merges away from the line with the source radiation, which takes over and is out-going sufficiently far from the line. Although there is no critical cone in this case the situation is not unlike that of the leaky wave in that the surface wave merges with the source radiation, which takes over at large distances and satisfies the radiation condition at infinity.

The question of whether a leaky wave is a "genuine" mode of a system is in part a semantic one. Obviously it can exist only in a limited space region because of the radiation condition at infinity. But one can also ask whether it is physically real in another sense - does it satisfy time-causal conditions? If a wave is switched on, does the leaky-wave component appear with a time-transient within the light-cone, or is there a precursor outside the light cone? In either case, can this be discerned from the propagating properties only of the wave? Do the space-varying characteristics imply also the time-characteristics? One would suspect so, though no such relationship appears to have been reported. The problem is complicated by a further feature of the leaky wave; its characteristics are frequency-dependent. In particular the critical angle approaches zero at high frequencies. Since all frequencies are developed when switching on there is a real problem in sorting out what constitutes the leaky mode during the initial period. As already mentioned, the leaky mode cannot exist in isolation and relating it to a source is essential.

In the present study the matter is examined from the point of view of a realistic system. A source and surface-wave structure is examined first under harmonic excitation, and the surface wave component is extracted by inspection following a contour deformation valid within the critical cone. This is sufficient to enable the surface wave to be recognized and its amplitude, as a function of frequency, to be determined in relation to the excitation. A Fourier-time analysis, on the surface-wave component only, is performed and the transient/precursor properties extracted.

It is found that both the leaky-wave and the Goubau-wave, in the example treated, are accompanied by a precursor outside the light-cone, so these waves, in isolation, do not satisfy the time-causal relation. What the significance of this may be is outside the scope of the present study. It does appear, however, that this is possibly a general property, and somehow might be deducible more directly for any mode not satisfying the radiation conditions, without having to make reference to a particular source problem.

1. Surface Wave Generation

In order to provide a realistic generation process, we consider a symmetrical center-fed cylindrical antenna of infinite length with a suitable surface impedance loading. The generator is located over a short distance 2δ , where δ can be taken later to zero since no divergencies arise from using a δ-function generator in this problem. The arrangement is shown in figure 1 and we define

$k_o = 2\pi/\lambda_o = \omega_o/c$ (free space propagation consant)

$k = 2\pi/\lambda$ (medium propagation constant, differing from k_o by possessing a small loss-factor. We can put $k = k_o\varepsilon_r^{\frac{1}{2}}$ with $\varepsilon_r \to 1$ in the limit of no loss)

ω_o = angular frequency of the wave.

$\zeta_o = (\mu_o/\varepsilon_o)^{\frac{1}{2}}$ (impedance of free space)

$\zeta = (\mu_o/\varepsilon)^{\frac{1}{2}} = \zeta_o/\varepsilon_r^{\frac{1}{2}}$ (impedance of the medium)

$c = (\mu_o\varepsilon_o)^{-\frac{1}{2}}$ (light velocity in vacuum)

Z_s = surface impedance of the cylindrical surface.

For an inductive surface, Z_s should vary proportional to ω_o and for a capacitive surface inversely proportional thereto. Thus, using ζ_o for normalization, we take

$$Z_s = jk_o\zeta_oL \qquad \text{(inductive surface)} \tag{1}$$

$$Z_s = -j\zeta_o/k_oC \qquad \text{(capacitive surface)} \tag{2}$$

L and C are frequency-independent parameters that determine the strength of the loading; they have the dimensions of length. Loading with inductance gives rise to a leaky wave whilst capacitive loading leads to a Goubau wave.

We shall examine first the properties for excitation by a time harmonic wave with time vector $e^{+j\omega t}$. The electric field component E_z can be taken as a Fourier integral of elementary cylindrical waves in the following form to satisfy the radiation condition as $\rho \to \infty$:

$$E_z = \int_{-\infty}^{\infty} H_o^{(2)}(\beta\rho)e^{-j\alpha z}F(\alpha)d\alpha \tag{3}$$

The variable α is real, $F(\alpha)$ is to be determined, and β is given by

$$\beta = (k^2-\alpha^2)^{\frac{1}{2}} \sim -j|\alpha| \qquad \text{for } |\alpha| \gg |k| \tag{4}$$

There are branch-points at $\alpha = \pm k$, and the integration contour is shown in figure 2.

The above form ensures that the radiation condition is satisfied for large ρ.

At $\rho = a$, the wire radius, we need

$$\begin{aligned} E_z + Z_s H_\theta &= 0 , \qquad |z| > \delta \\ &= -V_o/2\delta , \quad |z| < \delta \end{aligned} \tag{5}$$

The voltage V_o of the generator is arbitrary, and it is convenient to give it the value $-2\pi\zeta/jk$ to avoid unwanted constant factors later on.*

The form of H_θ follows from Maxwell's equations and is

$$H_\theta = (jk/\zeta)\int_{-\infty}^{\infty} \beta^{-1} H_1^{(2)}(\beta\rho)e^{-j\alpha z}F(\alpha)d\alpha \tag{6}$$

Writing, for a moment, (1) and (2) in the more general form $Z_s = j\zeta_o X$, the condition (5) can be written

$$\begin{aligned} \int_{-\infty}^{\infty} F(\alpha)e^{-j\alpha z}[H_o^{(2)}(\beta a) - kX\zeta_o H_1^{(2)}(\beta a)/\zeta\beta]d\alpha &= \pi\zeta/jk\delta , \quad |z| < \delta \\ &= 0, \quad |z| > \delta \end{aligned} \tag{7}$$

The solution of this integral equation is

---

*Actually the second equation should be written

$$E_z + Z_s H_\theta = -V_o/2\delta + (Z_s H_\theta)_{z=0} , \qquad |z| < \delta \tag{5a}$$

where it is assumed that the generator current, proportional to H_θ, is constant over the short range $0 < |z| < \delta$. However, this form only affects the arbitrary constant multiplier that enters into the problem and is not needed unless the field is to be referred to an actual generator. And in such a case the effective value of Z_s at the generator surface would also need to be known. The effect of the additional term on the right of (5a) becomes negligible as $\delta \to 0$ so that (10) is in any case correct for a δ-function generator.

$$F(\alpha)[H_0^{(2)}(\beta a) - kX\zeta_0 H_1^{(2)}(\beta a)/\zeta\beta] = \frac{1}{2\pi}\int_{-\delta}^{\delta} e^{j\alpha z} d\alpha(\pi\zeta/jk\delta)$$

$$= (\zeta/jk)\ \frac{\sin \alpha\delta}{\alpha\delta} \quad (8)$$

Substituting into (6) gives

$$H_\theta = \int_{-\infty}^{\infty} \frac{H_1^{(2)}(\beta\rho)e^{-j\alpha z}\ \frac{\sin \alpha\delta}{\alpha\delta}\ d\alpha}{\beta H_0^{(2)}(\beta a) - (k^2X/k_0)H_1^{(2)}(\beta a)} \quad (9)$$

Convergence of this integral does not require the presence of the factor $\sin \alpha\delta/\alpha\delta$, so it is convenient at this point to take $\delta = 0$ and (9) simplifies to

$$H_\theta = \int_{-\infty}^{\infty} \frac{H_1^{(2)}(\beta\rho)e^{-j\alpha z}\ d\alpha}{\beta H_0^{(2)}(\beta a) - (k^2X/k_0)H_1^{(2)}(\beta a)} \quad (10)$$

We shall work with this form in the rest of this study. It gives the magnetic field radiated from and guided along a δ-function-fed, symmetrical, cylindrical, surface-impedance structure.

2. Surface Wave Component

If the denominator of the integrand in (10) is equated to zero, there is found a root $\alpha = \alpha_0$ which gives rise to a discrete wave conducted along the antenna surface. By deforming the contour* as in figure 3 we obtain a residue at the resulting pole of amount

$$H_{\theta(\text{surface wave})} = H_1^{(2)}(\beta_0\rho)e^{-j\alpha_0 z}\, 2\pi j \text{ Residue } [\beta H_0^{(2)}(\beta a) - (k^2X/k_0)H_1^{(2)}(\beta a)]^{-1}\Big|_{\alpha = \alpha_0}$$

$$= (-2\pi j\beta_0/\alpha_0)H_1^{(2)}(\beta_0\rho)e^{-j\alpha_0 z}[H_0^{(2)}(\xi)(1-p) - H_1^{(2)}(\xi)(\xi - p/\xi)]^{-1} \quad (11)$$

---

*The deformation is permissible only within the critical cone for the leaky wave: this is sufficient to locate uniquely the leaky wave component. It is assumed that $z > 0$; results for $z < 0$ are obtained by replacing z by $-z$, because of the symmetry.

where

$$\beta_o = (k^2 - \alpha_o^2)^{\frac{1}{2}}$$

$$\xi = \beta_o a$$

$$p = k^2 Xa/k_o$$

Of course, (11) does not represent the major part of the field except in the region where the surface wave dominates. As discussed earlier, for z large enough the surface wave is attenuated to a value smaller than the direct source radiation so that in that region the remaining integration along the branch cut represents this radiated field. But in the region where it is significant, (11) represents the surface wave, with amplitude given by the residue expression. It involves a surface impedance parameter p, and in view of the presence of the factor ka we can assume that this parameter is small, at least for small wire radius and light loading.

The pole location α_o is given, via $\xi = \beta_o a$, as a root of the equation

$$\xi H_o^{(2)}(\xi) = pH_1^{(2)}(\xi) \tag{12}$$

An approximate value of ξ for small p can be found by arguing as follows: the equation cannot possess a root for large ξ since $H_1^{(2)}(\xi) \sim jH_o^{(2)}(\xi)$ for $|\xi| >> 1$. For $\xi = O(1)$ the right hand side is $O(p)$ whilst the left-hand side is $O(1)$. Hence any root must occur for small ξ. Since $H_1^{(2)}(\xi)$ varies as $1/\xi$ for small ξ, whilst $H_o^{(2)}(\xi)$ is logarithmic, a value of ξ very approximately of order $p^{\frac{1}{2}}$ must exist. By expanding the Hankel functions for small ξ we get the approximate equation

$$\xi^2[j\pi + 2\gamma + 2\log(\xi/2)](1-p/2) = -2p \tag{13}$$

For p small and positive (13) possesses a root close to

$$\xi_o = \left[\frac{2p}{\log(2/p) - 2\gamma - j\pi}\right]^{\frac{1}{2}} \tag{14}$$

This expression possesses a positive imaginary part, so that the exponential factor in $H_o^{(2)}(\beta_o\rho)$, i.e. $e^{-j\xi_o\rho/a}$ is increasing with ρ; a known property of a leaky wave on an inductive surface. For p small and negative (capacitive loading) the value of ξ_o is

$$\xi_o = -j\left[\frac{-2p}{\log(-2/p) - 2\gamma}\right]^{\frac{1}{2}} \tag{15}$$

This gives the expected radial attenuation of a Goubau surface wave over a capacitive surface. If the capacitive loading is slightly lossy, say due to the use of a dielectric with loss tangent δ, (15) becomes

$$\xi_o = -j\left[\frac{|-2p|}{\log|-2/p| - 2\gamma + j\delta}\right]^{\frac{1}{2}} e^{-j\delta/2} \tag{16}$$

The effect of the loss is to give a small negative real part to ξ_o, and since the asymptotic form of the Hankel function in (11) involves $\exp[-j\xi_o\rho/a]$ the loss gives rise to an incoming radial component to the wave, a well-known property of the Goubau wave.

It should be stressed that the approximations (14) and (15) can be used only if p is small enough; otherwise resort must be had to the basic equation (12). However, the only use made here of these relations is to note that there is indeed one root existing when p is small - the exact value is of no special consequence. (When p is large the only root is $\xi \sim jp$; although the properties of (12) in the whole complex plane have not been thoroughly explored it would appear that there is one, and only one, root existing for all relevant values of p.)

3. Time-Causal Properties: Leaky Wave

A wave of angular frequency ω_o switched on from zero to unit amplitude at $t = 0$ can be expressed by

$$f(t) = \frac{1}{2\pi j} \lim_{\Delta \to 0+} \int_{-\infty}^{\infty} \frac{e^{j\omega t}}{\omega - \omega_o - j\Delta} \, d\omega \tag{17}$$

The effect of the small positive quantity Δ is to place the pole at $\omega = \omega_o + j\Delta$ just above the real axis. For $t > 0$ convergence of the integrand permits the contour to be closed by the semi-circle at infinity in the upper half of the complex ω-plane, giving a single residue at $\omega = \omega_o + j\Delta$, resulting in $f(t) = e^{j\omega_o t}$ as $\Delta \to 0$. Similarly, for $t < 0$ the semi-circle needs to be in the lower half-plane for convergence. There is no pole there, so $f(t) = 0$ for $t < 0$. Clearly poles in the upper half-plane are associated with realistic transients or oscillations, whilst singularities in the lower half-plane would be associated with non-causal precursors. It is our aim to use (17) in conjunction with (11) to see where (11) - taken in isolation - exhibits singularities in the complex ω-plane, and hence to determine the causal properties of the leaky or surface waves. To do this it is necessary to restore the missing time vector $e^{j\omega_o t}$ in (11), replace ω_o by ω everywhere, and then apply to it the integral operator of (17), i.e. $\frac{1}{2\pi j} \lim_{\Delta \to 0} \int_{-\infty}^{\infty} \frac{(\ )\, d\omega}{\omega - \omega_o - j\Delta}$.

(Note that ω is contained implicitly in (11) through $k_o = \omega/c$.)
The effect is that of an isolated leaky-wave of angular frequency ω_o, switched on at $t = 0$. It follows that the singularities of (11), qua ω, determine the time-causal properties of the leaky wave. In particular, any singularities in $\text{Im}\, \omega < 0$ will give rise to non-causal properties.

In order to determine the general nature of these properties, it is not necessary to carry out the indicated integrations; it is sufficient to locate the singularities of (11). Clearly there are three possible locations to be examined; they are at i) $\beta_o = 0$ where the Hankel functions exhibit a branch-point, ii) $\alpha_o = 0$, where (11) exhibits a pole, and iii) at roots of $H_o^{(2)}(\xi)(1-p) = H_1^{(2)}(\xi)(\xi-p/\xi)$. We shall examine these roots in turn, bearing in mind that they must also satisfy (12), which locates the pole giving rise to the wave. In other words we have sets of two equations to be solved simultaneously, (12) in conjunction with each of the three preceding conditions in turn.

1). $\beta_o = 0$. Since, by definition, $\xi = \beta_o a$ and $a \neq 0$, $\beta_o = 0$ requires $\xi = 0$, and this is not compatible with (12). Hence the position $\beta_o = 0$ does not give an acceptable location of a singularity in the ω-plane.

2). $\alpha_o = 0$. From the definition of β_o in terms of α_o, this condition leads to $\xi = ka$ and (12) becomes, on using (1) for Z_s,

$$H_0^{(2)}(ka) = kLH_1^{(2)}(ka) \tag{18}$$

If we put $k = \omega/c$ (the distinction between k and k_o is of no relevance here), and $\omega = -j\phi$ the equation reduces to

$$K_o(\phi a/c) = (\phi L/c)K_1(\phi a/c) \tag{19}$$

Now as $\phi \to 0$, $K_o \to \infty$ whilst ϕK_1 is finite. Hence the left-hand side of (19) exceeds the right-hand side for small ϕ. As $\phi \to \infty$ both K_o and K_1 go to zero, but their ratio $\to 1$. Due to the presence of the factor ϕ, the right-hand side exceeds the left for large ϕ. Both functions in (19) are monotonic. Therefore there is one, and only one, real, positive value of ϕ which satisfies the equation. (It is not known for certain whether there are additional complex values. However, for k large the asymptotic relation $H_1^{(2)}(ka) \sim jH_o^{(2)}(ka)$ leads to the single root $\omega \sim -jc/L$, and there is similarly only a single root, found from the small argument expansions, when k is small. It therefore seems very likely that there is indeed only the one root. But the presence of further roots would not alter the subsequent conclusions).

3). $H_0^{(2)}(\xi)(1-p) = H_1^{(2)}(\xi)(\xi-p/\xi)$. This has to be solved simultaneously with (12), i.e. $\xi H_o^{(2)}(\xi) = pH_1^{(2)}(\xi)$, with $p = k^2La$. Eliminating the Hankel functions from this pair of equations gives $\xi^2 = 2p-p^2$, or $p = 1 - (1-\xi^2)^{\frac{1}{2}}$. Substituting into (12) gives

$$\xi H_o^{(2)}(\xi) = [1 - (1-\xi^2)^{\frac{1}{2}}]H_1{}^{(2)}(\xi) \tag{20}$$

If we put $\xi = -j\eta$ the equation takes the form

$$K_1(\eta) = K_o(\eta)[(1+\eta^2)^{\frac{1}{2}} + 1]/\eta \tag{21}$$

The only root of this equation appears to be at $\eta = \infty$, where the two sides become asymptotically equal. There is no real root, and apparently no complex root either, although without an exhaustive search it is difficult to be quite certain. However a finite root of (21), if it should exist, will not affect subsequent conclusions.

The time-causal properties are determined by the existence of a root coming from $\alpha_o = 0$. This gives $\omega = -j\phi$ with ϕ real and positive, and determined by (19). Since this root is in the region $\text{Im}\,\omega < 0$ it gives rise to a time precursor varying as $e^{\phi(t-\rho/c)}$ for $t < \rho/c$. The meaning of this conclusion is discussed further in section 5.

4. Time-Causal Properties: Goubau Surface-Wave

The analysis proceeds almost exactly as in section 3 except that k_oL is replaced by $-1/k_oC$. Instead of (18) we get

$$kCH_o^{(2)}(ka) = -\varepsilon_r H_1^{(2)}(ka) \tag{22}$$

in which ε_r can now be replaced by 1 and k by k_o.

Writing $\omega = k_oc = -j\phi$ the equation reduces to

$$\phi K_o(\phi a/c) = (c/C)K_1(\phi a/c) \tag{23}$$

For ϕ small the left-hand side is less than the right, whilst the opposite is the case for ϕ large. Both sides are monotonic, and there is therefore one real root for ϕ . Since $\omega = -j\phi$ we again see that there is a non-causal precursor, exactly as in the leaky wave case. Only the value of ϕ is different.

The consideration of a possible further root follows the identical analysis of section 3, since (20) is not affected by the change, though the related value of p is different. Accordingly we find that the Goubau wave behaves quite similarly to the leaky wave in its time-causal characteristics.

5. Discussion and Conclusions

Both the leaky wave and the Goubau surface wave, taken in isolation, i.e. without consideration of other fields that would be present when generated by a finite source, appear to contravene the radiation condition at infinity; the former because of its increasing fields outside the critical cone, and the latter because of its incoming character to supply the power lost on the guiding structure. It can therefore be asked in what sense these modes are genuine ones. The present study shows that, in the particular source example considered, they are non-causal. Is this a general property or one predicated on the particular feed system used? If it is a general one, then it would be expected that it could be deduced from other considerations, e.g. the form taken by the propagation factors. But apart from not satisfying the radiation conditions these factors do not seem to demand any particular time structure, though it may be speculated that there is a connection. However, the present method indicates one feature that may be of a general character. The non-causal nature comes from the pole at $\alpha_o = 0$, the presence of the latter arising from a differentiation of β, since $\partial\beta/\partial\alpha = -\alpha/\beta$. The β appears here from the radial term via $H_o^{(2)}(\beta a)$, and since the source matching always occurs at some value or values of ρ, a boundary condition involving β seems inevitable. Hence we would always expect a pole at $\alpha_o = 0$, whatever other features might be in evidence. This pole leads to a space variation from $H_o^{(2)}(\beta\rho)e^{-j\alpha z}$ of $e^{-jk\rho}/\rho^{\frac{1}{2}}$ for large ρ , all z. In the present example this term is damped for large ρ, when $\omega = -j\phi$. If it is physically meaningful to require that this term always be damped then we require $k \propto e^{-j\psi}$ with $0 < \psi < \pi$. This leads to a pole for ω in the lower-half plane, and hence to a precursor varying as $e^{\psi(t-\rho/c)}$ for $t < \rho/c$, i.e., outside the usual light cone. This result might possibly be related to the radiation condition in the following way. A decaying pulse $e^{-t/\tau}$ can be represented by a real spectrum via the equation

$$e^{-t/\tau} = \frac{2\tau}{\pi}\int_0^{\infty} \frac{\cos \omega t}{1 + \omega^2\tau^2} d\omega \qquad (24)$$

Since the fields of the leaky wave increase exponentially in the radial direction for real frequencies the relation (24) would indicate a similar property of a decaying pulse. The corresponding angular frequency in complex notation would be positive imaginary, or $k \propto e^{+j\psi}$, $\psi = \pi/2$. To get a physically meaningful decaying radial field it would be necessary to reverse the sign of ψ, leading to the criterion hypothesized above.

The conclusion to be drawn from this somewhat speculative assessment is that isolated surface modes of an open system are not time-causal. The total field, of course, is time-causal, and this throws some doubt on the validity of working with isolated surface modes, their utility in antenna design notwithstanding.

It might be objected that in the above reasoning the location $\alpha_o = 0$ is always outside the critical cone, and therefore in a region of no practical interest for the leaky wave. Neither is it encountered, for real frequencies, with the (slow) Goubau wave. The trouble here is that the structure of these waves is frequency-dependent. The Goubau wave extends to infinity at low frequencies whilst the leaky wave has a critical angle of zero at high frequencies. Since all frequencies are generated by switching on it becomes unclear how the transient characteristics at initiation of a leaky wave in isolation should be interpreted. In practice, of course, the total field builds up in a sensible way and "as the dust settles," the leaky wave aspect emerges in that region where the field can be meaningfully interpreted in terms of it. It is only because of a wish to deal with the isolated mode that the problem arises. One needs to know, for example, whether a predicted mode is a genuine characteristic that can be used for transmission of information, or is merely a mathematical artifact with no real useful properties. It is hoped that the results of this study may eventually throw some light on this.

Acknowledgments. The author wishes to express his thanks to David Chang for helpful discussion on many of the points raised.

References

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2. H.M. Barlow and A.L. Cullen, "Surface Waves," J,I.E.E. Part 3, Nov. 1953, pp. 329-341.

3. *Ibid.*, p. 344.

Figure Captions

1. Antenna and Source

2. Contour in the α- plane

3. Deformed contour in the α-plane.

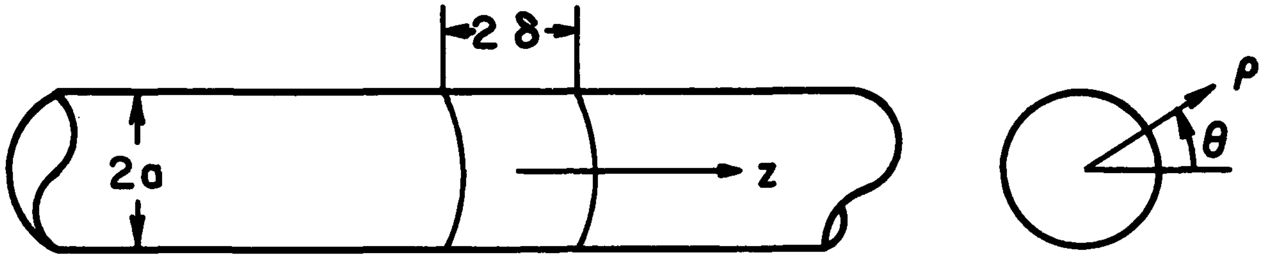

Figure 1

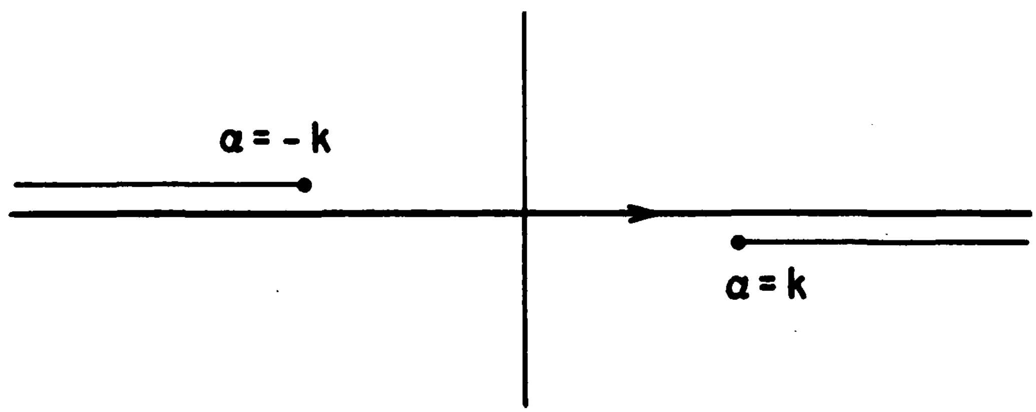

Figure 2

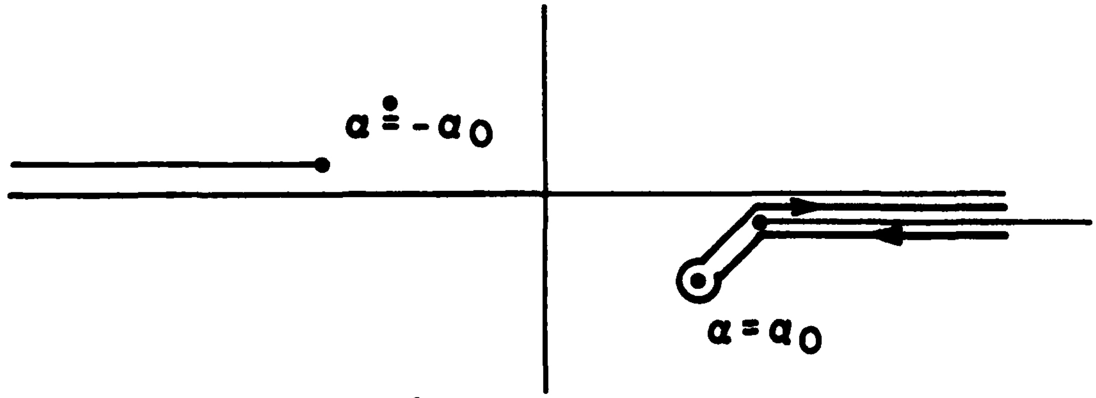

Figure 3